Lecture 9: Neutron Interactions and Cross Sections

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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1 Introduction: The Shift to Induced Reactions

Primary Reference: Lamarsh, 4th Ed., Section 3.1

In the first phase of this course, we studied *spontaneous* nuclear transformations—radioactive decay ($dN/dt = -\lambda N$). We are now shifting to *induced* reactions, where we actively bombard nuclei with projectiles to force a transformation.

While alpha particles and protons were used in early experiments, they suffer from a major limitation: the **Coulomb Barrier**. To enter a nucleus, a charged particle must possess enough kinetic energy to overcome the electrostatic repulsion of the target protons.

Neutrons, being neutral, face no such barrier. A neutron with negligible kinetic energy ($E \sim 0.025$ eV) can wander into a uranium nucleus as easily as a fast neutron ($E \sim 2$ MeV). This property makes the neutron the ideal agent for nuclear engineering.

2 The Microscopic Cross Section (σ)

Primary Reference: Lamarsh Section 3.2

When a neutron approaches a nucleus, the probability of an interaction depends on the effective "target area" presented by the nucleus. We call this the **Microscopic Cross Section**, denoted by σ.

2.1 The Barn

The standard unit for cross sections is the **barn** (b).

$$1 \text{ b} = 10^{-24} \text{ cm}^2 \tag{1}$$

This value is roughly the geometric cross-sectional area of a medium-sized nucleus. However, the *interaction* cross section is a quantum mechanical property and can be vastly different from the geometric size.

3 Classification of Interactions

Reference: Lamarsh Section 3.1 - 3.2

A neutron does not just "hit" a nucleus; it interacts via specific channels. The **Total Cross Section** (σ_t) is the sum of the probabilities of all possible distinct events.

$$\sigma_t = \sigma_s + \sigma_a \tag{2}$$

3.1 1. Scattering (σ_s)

The neutron interacts with the nucleus and re-emerges, potentially with lower energy.

- **Elastic Scattering (σ_e or (n, n)):** "Billiard ball" collision. Kinetic energy is conserved in the center-of-mass system. This is the primary mechanism for slowing down neutrons in a moderator (e.g., Water, Graphite).
- **Inelastic Scattering (σ_i or (n, n')):** The neutron strikes the nucleus and leaves it in an excited state. The neutron loses significant kinetic energy. This generally requires high-energy neutrons (Fast).

3.2 2. Absorption (σ_a)

The neutron enters the nucleus and does not re-emerge as a neutron.

- **Radiative Capture (σ_γ or (n, γ)):** The neutron is captured, and the nucleus emits a gamma ray. (Example: $^{238}\text{U} + n \rightarrow ^{239}\text{U} + \gamma$).
- **Fission (σ_f or (n, f)):** The nucleus splits into two large fragments plus 2–3 neutrons.
- **Charged Particle Emission (σ_p, σ_α):** Example: $^{10}\text{B} + n \rightarrow ^{7}\text{Li} + \alpha$ (Control Rods).

4 The Macroscopic Cross Section (Σ)

Primary Reference: Lamarsh Section 3.3

In engineering, we deal with bulk matter. We define the **Macroscopic Cross Section** (Σ) to describe the interaction probability per unit volume.

$$\Sigma = N\sigma \tag{3}$$

where N is the atom density (atoms/cm^3). The unit of Σ is $\mathbf{cm^{-1}}$, representing the probability of interaction per cm of travel.

4.1 Calculating Atom Density (N)

For a pure substance with density ρ (g/cm^3) and atomic weight M (g/mol):

$$N = \frac{\rho N_A}{M} \tag{4}$$

For a mixture (like H_2O), the macroscopic cross section is additive:

$$\Sigma_{mix} = \sum_i N_i \sigma_i \tag{5}$$

Example 1: Macroscopic Cross Section of Water

Problem: Calculate the total macroscopic cross section (Σ_t) for liquid water at 20°C. Assume: $\rho = 1.0$ g/cm^3; $\sigma_t^H = 21.0$ b; $\sigma_t^O = 4.0$ b.

Solution: 1. Calculate Molecular Density of Water (N_{H2O}):

$$N_{H2O} = \frac{\rho N_A}{M} = \frac{(1.0)(0.6022 \times 10^{24})}{18.0} = 0.0335 \times 10^{24} \text{ molecules/cm}^3$$

2. Determine Atom Densities (N_H and N_O):

$$N_O = 1 \times N_{H2O} = 0.0335 \times 10^{24} \text{ atoms/cm}^3$$

$$N_H = 2 \times N_{H2O} = 0.0670 \times 10^{24} \text{ atoms/cm}^3$$

3. Calculate Σ_t:

$$\Sigma_t = N_H \sigma_t^H + N_O \sigma_t^O$$

$$\Sigma_t = (0.0670)(21.0) + (0.0335)(4.0) \quad [\text{units of } 10^{24} \cdot 10^{-24} \text{ cancel}]$$

$$\Sigma_t = 1.407 + 0.134 = \mathbf{1.54} \text{ cm}^{-\mathbf{1}}$$

Note: Hydrogen contributes over 90% of the interaction probability. The collision cross section listed is for free hydrogen and appropriate for higher energy neutrons. For low energy neutrons the collision cross section for hydrogen bound up in water is five times greater.

5 Beam Attenuation and Mean Free Path

Consider a collimated beam of neutrons with intensity I_0 incident on a slab. The removal of neutrons follows Beer's Law:

$$I(x) = I_0 e^{-\Sigma_t x} \tag{6}$$

The average distance a neutron travels before interacting is the **Mean Free Path** (λ):

$$\lambda = \frac{1}{\Sigma_t} \tag{7}$$

Example 2: Shielding Calculation

Problem: An experiment requires a neutron beam to be shielded by a slab of Indium ($\Sigma_a = 4.8 \text{ cm}^{-1}$). How thick must the slab be to reduce the beam intensity by 95%?

Solution: If intensity is reduced *by* 95%, the remaining intensity $I(x)$ is **5%** of I_0.

$$\frac{I(x)}{I_0} = 0.05$$

$$0.05 = e^{-\Sigma x} \implies \ln(0.05) = -\Sigma x$$

$$-2.996 = -4.8x \implies x = \frac{2.996}{4.8} = \mathbf{0.624} \text{ cm}$$

6 Reaction Rates and Flux

Primary Reference: Lamarsh Section 3.4

In a reactor, neutrons move randomly in all directions. We define the **Scalar Flux**, $\phi = nv$, where n is neutron density and v is speed. The **Reaction Rate Density** (R) is the fundamental link between physics and engineering:

$$R = \phi \Sigma \tag{8}$$

(Units: interactions $\cdot \text{cm}^{-3} \cdot \text{s}^{-1}$)

Example 3: Measuring Flux with Gold Foil

Problem: A gold foil (mass 1.0 mg) is irradiated in a reactor. The foil becomes radioactive at a rate of 10^8 interactions/sec (R_{total}). Calculate the neutron flux ϕ. Assume: $\rho_{Au} = 19.3$ g/cm^3; $M_{Au} = 197$ g/mol; $\sigma_\gamma = 98.7$ b.

Solution: The total reaction rate in the foil is $R_{total} = \phi(\text{Total Atoms})\sigma_\gamma$. 1. Calculate Total Atoms:

$$\text{Atoms} = \frac{mN_A}{M} = \frac{(0.001)(0.6022 \times 10^{24})}{197} = 3.057 \times 10^{18} \text{ atoms}$$

2. Solve for ϕ:

$$10^8 = \phi \cdot (3.057 \times 10^{18}) \cdot (98.7 \times 10^{-24})$$
$$10^8 = \phi \cdot (3.017 \times 10^{-4})$$
$$\phi = \mathbf{3.31 \times 10^{11}} \text{ neutrons/cm}^{\mathbf{2}}\text{s}$$

7 Resources for Visualization

1. **NNDC Sigma Plotter:** To see the difference between Capture (σ_γ) and Fission (σ_f) cross sections for U-235:
   https://www.nndc.bnl.gov/sigma/ (Select "n,g" for capture and "n,f" for fission).
2. **MIT OpenCourseWare (22.01):** Lecture 21 covers Neutron Interactions.
   https://ocw.mit.edu/courses/22-01-introduction-to-nuclear-engineering-and-ionizing-radiat pages/syllabus/
3. **HyperPhysics:** Cross Section Concepts.
   http://hyperphysics.phy-astr.gsu.edu/hbase/Nuclear/crosec.html

How to use the NNDC Sigma Plotter

The National Nuclear Data Center (NNDC) provides the standard evaluated cross-section libraries (ENDF/B) used by nuclear engineers.

1. **Access the Tool:** Go to https://www.nndc.bnl.gov/sigma/.
2. **Method A: Visual Search (The Periodic Table)**
   - Click on the element symbol in the Periodic Table (e.g., **U** for Uranium).
   - A list of isotopes will appear. Click the specific isotope mass number (e.g., **235**).
   - A table of "Reactions" will appear.
3. **Method B: Direct Search (Faster)**
   - Select the "Basic Retrieval" tab.
   - In the search bar ("Target"), simply type the isotope name (e.g., `U235` or `Fe56`).
   - **Selecting Reactions:** Select the checkboxes for the cross sections you wish to compare:
     - **(n,tot):** Total Cross Section (σ_t)
     - **(n,el):** Elastic Scattering (σ_s)
     - **(n,g):** Radiative Capture (σ_γ)

  - **(n,f):** Fission (σ_f)
- Click on "Submit"

4. **Plotting:** Click the **Plot** button. Use the axis settings to switch to Log-Log scales for better visibility of resonances.

Recommended Exercise: The "Iron Window"

To understand why shielding calculations are difficult, try this:

- Search for `Fe-56` (the main isotope in steel).
- Plot the Total Cross Section `(n,tot)`.
- **Observation:** Look at the graph around **24 keV**. You will see a deep "valley" where the cross section drops precipitously (from ∼10 b down to ∼0.1 b).
- **Significance:** This is known as a "Neutron Window." At this specific energy, neutrons can stream through thick steel shields almost as if they were transparent. This effect (interference minimum) makes Iron-56 a complex material for shielding fast neutrons.